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A leverage-based measure of financial stability☆

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The size and the leverage of financial market investors and the elasticity of demand of unlevered investors define MinMaSS, the smallest market size that can support a given degree of leverage. The financial system's potential for financial crises can be measured by the *stability ratio*, the ratio of total market size to MinMaSS. We use that financial stability metric to gauge the buildup of vulnerability in the run-up to the 1998 Long-Term Capital Management crisis and argue that policymakers could have detected the potential for the crisis.

1. Introduction

The timing of financial crises is difficult to predict as crises are triggered by the realization of adverse shocks which tend to be unforeseeable. However, the financial systems' vulnerability to adverse shocks is measurable. When system vulnerability is high, shocks can trigger adverse feedback loops via amplification mechanisms such as leverage spirals. Hence financial stability monitoring efforts focus on measuring the degree of financial vulnerability by gauging the evolution of amplification mechanisms in the financial system (see Adrian et al., 2015).

In this paper, we study an important amplification mechanism for financial crises based on the leverage cycle of financial market investors. We develop a theoretical setting of leveraged and unleveraged financial market investors to derive a metric of aggregate leverage that gives rise to a quantitative condition for stability. The stability condition can be evaluated from observable characteristics and can give policymakers advance warning of financial crises.

We define a financial market equilibrium as unstable when the process of *tatonnement* pushes the system away from, rather than towards, equilibrium (Hahn, 1982). This corresponds to a situation where demand rises with price, and does so faster than supply. We focus on leverage of investors as a mechanism to generate upward sloping demand curves. When the proportion of levered investors is large, the *aggregate* demand curve for assets can become upward sloping, leading to an unstable equilibrium. Prices can thus exhibit discontinuity akin to a financial crisis. We develop a quantitative condition for such a market instability.

We show how this financial stability metric could have been used in the context of the 1998 Long-Term Capital Management (LTCM) crisis. That crisis is particularly relevant to our setting as it resulted from an interplay of leveraged financial market investors that resulted in abrupt price changes. In our application, we show that our calibrated stability ratios were not low enough to destabilize equity or Treasury markets, but that they could have been low enough to destabilize bank funding and equity volatility markets. The consequences of this potential

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instability eventually prompted the Federal Reserve to step in and coordinate a private-sector bailout.

Our modeling approach centers on leverage constraints. The related literature falls into four main categories. The first focuses on the impact of credit constraints to the real economy. Seminal contributions in this area are Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), who show that financial constraints can generate persistence and amplification of macroeconomic activity in response to negative net worth shocks. In contrast, our approach is more squarely focused on the role of the financial sector in asset price amplification, which could in turn be embedded in a macroeconomic setting.

A second strand of the literature evaluates the effect of collateral requirements in financial markets, showing how selling pressure from negative net worth shocks can amplify asset price fluctuations (Xiong (2001), Geanakoplos (2003), Yuan (2005), Adrian and Shin (2010), Gromb and Vayanos (2010), and Acharya and Viswanathan (2011)). Others have examined feedback loops when small shocks cause cascading liquidations through channels *other* than collateral constraints; these include fund redemptions (Shleifer and Vishny, 1997), price movements interpreted as fundamental signals (Diamond and Verrecchia, 1980; Gennotte and Leland, 1990), tightening margin requirements (e.g., Brunnermeier and Pedersen, 2009; Fostel and Geanakoplos, 2008), or uncertainty about bank solvency (Gorton and Metrick, 2012). All of the papers in this second strand of the literature present alternative microfoundations for leverage based amplification mechanisms. In contrast, we take as given that there are leveraged and unleveraged investors, and we simply ask what leverage constraints imply for equilibrium pricing. Hence our approach can be viewed as a reduced form representation of the alternative microfoundations of investor financial constraints. The advantage is simplicity.

Third, a growing literature evaluates systemic financial sector risk. Duarte and Eisenbach (2020) present a financial stability metric based on fire sale spillovers among investors, Capponi and Larsson (2015) study the impact of capital requirements on systemic market stability, and Brunnermeier and Cheridito (2019) offer an axiomatic metric of systemic risk. Our financial vulnerability metric MinMaSS falls squarely within this literature on systemic risk measurement, but our framework is simpler and more straightforward to implement. De Nicoló and Lucchetta (2011) and Giglio et al. (2016) evaluate the information content of alternative systematic risk metrics for macroeconomic activity.

Finally, another strand of the literature focuses on accounting rules as an amplification mechanism. Heaton et al. (2010) show that the impact of accounting rules on credit constraints can have a real macroeconomic consequence, and thus regulatory capital requirements should be adjusted for accounting rules. We leave the study of optimal capital regulation to future work, and note that our setting presumes mark-to-market accounting.

In sum, our proposed framework produces similar dynamics to other theories of leverage cycles, but we rely on only basic assumptions and present a particularly parsimonious and tractable approach. We make minimal behavioral assumptions. We do not rely on specific functional forms of preferences or shocks. Our approach is thus straightforward and less complex than alternative models proposed in the literature. This means that the model is very transparent about what is driving the results, and straightforward to implement empirically.

The model features constant leverage ratios, which are more tractable than endogenously tightening margin requirements. We show that even when margin requirements are constant and exogenous, an overly leveraged system is susceptible to crisis. This suggests that the amplification mechanisms associated with tightening margin constraints are additional amplifiers for deleveraging cycles, but such crises can occur even when leverage ratios are constant. As a result, policies that aim at mitigating the impact of increasing margins during volatile times may not be enough to address the fundamental deleveraging mechanism of crises. Even constant leverage ratios can trigger deleveraging cycles.

The model also permits investigation of contagion in a simple analytical framework, and shows that contagious instability is more likely to occur during crashes than during booms. Declining asset prices cause lower net worth, thus making leverage constraints more binding, and exacerbating the adverse feedback loop.

Section 2 develops a simple, intuitive version of the model. Section 3 fully generalizes the model to a degree that permits it to be calibrated. As part of the generalization, we show how to translate derivatives exposures into leverage metrics. We also show that when levered investors participate in multiple markets, a crash in one market can lead to contagion to other markets. Section 4 applies our proposed measure of financial stability to the 1998 collapse of hedge fund Long-Term Capital Management. Section 5 concludes.

2. The basic model

There are four types of agents: *levered investors*, who maximally leverage positions as permitted by their lenders; *fully funded investors*, who have a downward sloping demand curve for assets and who deposit any excess funds in a bank account; *banks*, who provide credit to the levered investors at the market interest rate and a fixed margin requirement (the reasons for these assumptions are discussed later); and a *central bank*, whose sole function is to hold interest rates fixed in the near term by providing credit to the market against sound collateral.

Our key assumption about the behavior of levered investors is that they are extremely enthusiastic about assets, so that purchases are leveraged to the maximum degree that lenders permit. Such seemingly simplistic behavior is assumed for tractability. This assumption can in fact be supported by appropriate microfoundations; the literature investigating optimal portfolio choice in the presence of net worth constraints and credit constraints finds that sufficiently optimistic rational agents do indeed employ leverage to the maximum degree permitted by their lenders (e.g., Grossman and Vila, 1992; Liu and Longstaff, 2004).[1]

As a result of these assumptions about levered investors' behavior, their net worth is then given by:

$$\text{Net Worth} = (\text{Margin Percentage})\cdot(\text{Assets}) \tag{1}$$

$$\equiv \lambda \cdot p_t m_t^{\text{lv}} \tag{2}$$

where p_t is the price of the asset at time t, m_t^{lv} is the quantity of the asset held by the levered investor at time t and λ is the margin requirement imposed by lenders or by regulators (the minimum proportion of the investor's assets that have to be covered by equity). A margin requirement of five percent (λ=0.05), for example, would indicate that at least five percent of the investor's assets have to be covered by equity.

Each period, Δt, the investor will reap the benefit of all price appreciation and dividends d_t from the assets and pay interest rate r_t charged on margin loans, so the change in net worth will be given by:

$$\Delta NW^{\text{lv}} = \text{Appreciation} + \text{Dividends} - (\text{Margin Interest}) \tag{3}$$

$$= (p_t - p_{t-1})m_{t-1}^{\text{lv}} + d_t \Delta t \cdot m_{t-1}^{\text{lv}} - (1-\lambda) r_t \Delta t \cdot p_{t-1} m_{t-1}^{\text{lv}} \tag{4}$$

Adding (2) at time $t-1$ and (4), and simplifying, we have:

$$NW_t^{\text{lv}} = m_{t-1}^{\text{lv}} \cdot [p_t + d_t \Delta t - (1-\lambda)(1 + r_t \Delta t) p_{t-1}] \tag{5}$$

This equation says simply that net worth is given by the current value of last period's assets, plus any dividends received on those assets, minus

[1] Even if real-world levered investors have some slack and cushion built in for the short term, investors still tend to target a certain leverage ratio over the medium term: they voluntarily liquidate when their net worth declines in order to avoid forced liquidations later, so they face what is effectively a "soft" margin requirement, see, for example, Shleifer and Vishny (1997)

the value of debt (with interest) funding those assets.

The levered investor invests her profits back into the asset with leverage. A combination of (2) and (5) simplifies to:

$$m_t^{\mathrm{lv}} = \frac{m_{t-1}^{\mathrm{lv}}}{\lambda} \cdot \left[1 + \frac{d_t \Delta t - (1-\lambda)(1 + r_t \Delta t) p_{t-1}}{p_t}\right] \tag{6}$$

This gives the levered investors' demand for assets m_t^{lv} as a function of the price p_t. The numerator within the bracket is almost certainly negative and hence the levered investors' demand for assets is upward sloping.[2] Investors targeting a specific leverage ratio demand more of an asset as its price increases. Finally, leverage introduces path-dependent demand: levered investors' demand depends on both yesterday's holdings and yesterday's price.

The remaining investors who invest without leverage are termed fully funded investors. These investors are not modeled in detail, but are assumed to have a downward sloping demand curve for assets. This downward sloping (rather than horizontal) demand may be for a variety of reasons, including heterogeneity of opinion about the value of the asset, relative value considerations, and the desire for portfolio diversification.[3] More simply, investors who eschew the use of leverage are limited in their asset purchases by their equity, and so the maximum number of shares they are able to purchase is a declining function of the share price. Demand by fully funded investors is then given by:

$$\begin{aligned}\text{Demand for Assets} = {} & (\text{Proportion of Fully Funded Investors}) \\ & \times(\text{Population of Investors}) \\ & \times(\text{Demand per Fully Funded Investor})\end{aligned}$$

$$\text{or: } m^{\mathrm{ff}} = (1-\mu) N \cdot D(p) \tag{7}$$

where m_t^{ff} is the total demand for assets by fully funded investors, μ is the proportion of investors that are levered, N is the total number of investors in the economy and $D(p)$ is the number of units of the asset that the average fully funded investor demands as a function of price. We shall assume that $D'(p) < 0$ so that demand is downward sloping and that demand does not depend upon the investor's net worth.[4]

Banks are assumed to be conduits that lend to all comers against collateral at the prevailing interest rate, which is fixed by a central bank, and with a fixed margin requirement. The existence of banks links the money supply to credit growth and hence links monetary policy to credit provided to speculative endeavors. The inclusion of banks is not a necessary condition for the model to work, but we include them nonetheless in order to ensure we meet adding-up constraints.

2.1. Model dynamics

The model's dynamics are governed largely by the behavior of the levered and fully funded investors. We shall investigate how the model behaves over short periods of time, where interest and dividend payments can be neglected.

The total demand for the asset, m_t, is the sum of the demand by levered and fully funded investors:

$$m_t = m_t^{\mathrm{ff}} + m_t^{\mathrm{lv}} \tag{8}$$

$$= (1-\mu)N \cdot D(p_t) + \frac{m_{t-1}^{\mathrm{lv}}}{\lambda} \cdot \left[1 - \frac{(1-\lambda)p_{t-1}}{p_t}\right] \tag{9}$$

It is clear that if $(1-\mu)N \cdot D(p)$ is large compared to m_{t-1}^{lv}, the fully funded investors will dominate and the demand curve will be downward sloping. However, if levered investors begin to do well, reinvest their proceeds and accumulate the asset, m_{t-1}^{lv} will begin to grow large relative to $(1-\mu)N \cdot D(p)$. Equation (9) determines when the system will exhibit explosive behavior. If its derivative is positive, at the previous period's equilibrium demand slopes upward. The system is unstable and a crash or price spike will result. If the derivative is negative, the system is stable. Let us examine this formally. Differentiating (9), we have:

$$\frac{dm_t}{dp_t} = (1-\mu)N \cdot D'(p_t) + \frac{(1-\lambda)}{\lambda} \frac{p_{t-1} m_{t-1}^{\mathrm{lv}}}{p_t^2} \tag{10}$$

If we know $D'(p)$, the demand response of the fully funded investors to small changes in price, we thus can determine whether the system is stable.

Define $A(p) \equiv (1-\mu)NpD(p)$ to be the total dollar amount fully funded investors in the aggregate wish to hold of the asset. Substituting into equation (10) and rearranging, the condition for stability ($\frac{dm_t}{dp_t} < 0$) then becomes:

$$\frac{NW_{t-1}^{\mathrm{lv}}}{\lambda^2} + (1-\eta_D)A(p_t) < p_{t-1} m_{t-1}^{\mathrm{lv}} + A(p_t) \tag{11}$$

where η_D (a positive number) is the fully funded investors' price elasticity of demand $-pD'(p)/D(p)$. The right hand side of the equation is the market size: the total assets held by levered investors, plus the total assets held by fully funded investors.

The left-hand side of equation (11) is the net worth of levered investors divided by the square of the margin requirement, plus the amount unlevered investors hold of the asset adjusted for their elasticity of demand. This quantity defines the *minimum market size for stability* of the market, which we shall term *MinMaSS*. It is the smallest market size that is consistent with stability; if $\eta_D = 1$ then MinMaSS is just the net worth of levered investors divided by the squared leverage ratio. We can form a ratio of market size to MinMaSS; we call this the stability ratio. If the *stability ratio* is greater than one, the market is stable; if it is less than one, the market is unstable. We can use the stability ratio in a straightforward way to determine how stable the market is. The closer is the stability ratio falls toward one, the closer is the market to becoming unstable.

Higher margin requirements support stability with a higher share of levered investors, while a relatively small number of levered investors can create instability if leverage is high. For example, suppose levered investors have a 10% margin requirement ($\lambda = 0.1$) and the elasticity of demand η_D is 1. Then levered investors will only need net worth of 1% of the total demand for the asset to create an unstable situation. This suggests that instability in markets may not be a particularly rare state of affairs. On the other hand, if the capital ratio is moderate, say 1:1, levered investors will need net worth of 25% of the total demand for the asset to create an unstable situation. A lower capital buffer is dangerous because, as equation (11) shows, MinMaSS goes as the *square* of the margin requirement λ.

When the stability ratio is below one, that is, when the market is unstable, small disturbances to the equilibrium lead to large price movements: there can be an immediate crisis, as the price crashes and levered investors go bankrupt, or there can be a price explosion. Once large price movements have occurred, it can no longer be taken for granted that levered investors continue to remain fully levered, but as

[2] The only way it is not is when both the time step between margin checks is large and the dividend is very large compared to the margin interest rate. To see how unlikely this is, consider the case where $\Delta t = 1$, that is, portfolio reallocations and margin calls take place only once a year. Consider a high capital ratio of 90% (nine dollars of equity for every dollar of debt), and a margin interest rate of just 1.5%. The dividend yield would then have to be greater than 10.15% in order for the demand curve to be downward sloping.

[3] Relative value is a method of determining an asset's value by considering the value of similar assets. The value investor invests less into the more overvalued asset, which puts downward pressure on its price. Portfolio diversification is when an investor wishes to keep a fixed proportion of her portfolio in different assets, such as the orthodox portfolio split of 60% stocks and 40% bonds. As the price of stocks rises, she needs to hold fewer shares to account for 60% of her portfolio.

[4] See Appendix A for discussion.

long as they do so we can continue to employ equation (11) to assess the stability of the market. However, it is important to note that the relevant margin requirement λ is the minimum leverage ratio required by financial regulators (or targeted by investors), rather than the actual margin requirement observed in the market.[5]

In summary, MinMaSS is determined by the characteristics and holdings of levered investors and by the demand curve of fully funded investors. Instability results from insufficient capital: If levered investors grow in the market, their perverse demand curves overwhelm the downward sloping demand from more prudent investors, eventually causing total demand to become upward sloping. This makes the typical equilibrium between supply and demand an unstable "knife edge" with no mechanism to force a convergence to that equilibrium. The lower capital ratio that is required, the more fragile the market in the sense that it takes a smaller share of levered investors to cause an unstable situation.

3. An operationalizable version of the model

We have so far restricted ourselves to a market with only one asset, no short selling, and only one class of levered investors, a choice made for expositional clarity. In this section, we expand the MinMaSS framework to incorporate markets where different investors lever to differing degrees, where investors sell short and take positions using derivatives, and with multiple assets, which will give rise to contagion. The resulting measure accounts for sufficient diversity that it might be used by macroprudential regulators as an early warning sign against financial crises.

Let us suppose that there are many assets, indexed by j, and that each asset has some derivative contracts associated with it, indexed by δ. Levered investors are indexed by i and have net worth NW_i^{lv}. Each asset has a *collateral value*, the maximum amount that can be borrowed against it, which may vary by investor. If the price of asset j at time t is p_t^j, then its collateral value for investor i is defined to be $(1-\lambda_i^j)p_t^j$. Each derivative contract δ on asset j must be also collateralized. At each time t, investor i allocates a proportion of her net worth $\pi_{it}^{j\delta}$ (which may be a function of the price vector $\mathbf{p_t}$) to collateralize each asset and derivative contract in which she invests. As above, we assume that levered investors are leverage constrained, meaning they use all their capital: $\sum_{j\delta}\pi_{it}^{j\delta}=1$.

Each investor's direct demand for the asset is given by the quantity she can buy with the share π_{it}^j of her net worth she devotes to that asset:

$$\text{Direct Demand}=\frac{\text{Capital Devoted to Asset}}{\text{Margin Requirement per Unit}}=\frac{\pi_{it}^j NW_{it}^{\mathrm{lv}}}{\lambda_i^j p_t^j}\equiv m_{it}^{jd} \tag{12}$$

With regard to derivatives, we consider contracts with single-period margining, meaning that any changes in fair value of the contract are paid or received each trading period. The most common examples of such contracts are exchange-traded futures and options contracts, although most credit default swaps and interest rate swaps have similar features.

The value of each derivative contract δ on asset j is a function of the price of the asset, which we shall denote $f_{j\delta}(p_t^j)$. (We treat a short sale as the special derivative contract where $f(p_t^j)=-p_t^j$.) For each investor i, asset j, derivative contract δ, and time t, we shall say that she holds $C_{it}^{j\delta}$ derivative contracts. For each of these contracts, she must post a fixed dollar amount of collateral $\chi_{it}^{j\delta}$ as *initial margin* with the exchange or counterparty.

We assume that levered investors may be on either the long or short side of the derivative contracts, so that C may be positive or negative. Of course, every derivative contract has two sides, so the net supply of derivative contracts must be identically equal to zero. Therefore, we introduce a *market maker* who is assumed to absorb any disparity in demand between long and short speculative positions and hedge these positions in the cash market. For each derivative contract, the position $C_{Ht}^{j\delta}$ held by the market maker (denoted by the subscript H to differentiate from investors' demand) is just the inverse of the net position of the levered investors:

$$C_{Ht}^{j\delta}=-\sum_i C_{it}^{j\delta} \tag{13}$$

In order to be hedged, a market maker wishes to be indifferent to price changes in the underlying asset. For each derivative contract δ, she therefore demands underlying assets $m_{Ht}^{j\delta}$ according to the condition:

$$\frac{\partial}{\partial p_t^j}\left[C_{Ht}^{j\delta}f\left(p_t^j\right)+m_{Ht}^{j\delta}p_t^j\right]=0 \tag{14}$$

or

$$m_{Ht}^{j\delta}=-C_{Ht}^{j\delta}f'\left(p_t^j\right)=\sum_i C_{it}^{j\delta}f'\left(p_t^j\right) \tag{15}$$

The behavior defined by equation (15) is known in financial markets as "delta hedging."[6] Each investor i makes a contribution to the market maker's delta hedging activities for derivative contract δ in proportion to the investor's holdings. It therefore makes sense to refer to this contribution as the investor's indirect demand for the asset:

$$\text{Indirect Demand}=C_{it}^{j\delta}f_{j\delta}'\left(p_t^j\right)=\sum_\delta\frac{\pi_{it}^{j\delta}NW_{it}^{\mathrm{lv}}}{\chi_{it}^{j\delta}}\cdot f_{j\delta}'\left(p_t^j\right) \tag{16}$$

Investor i's total effective demand for asset j is the sum of direct and indirect demand:

$$m_{it}^j=\frac{\pi_{it}^j NW_{it}^{\mathrm{lv}}}{\lambda_i^j p_t^j}+\sum_\delta\frac{\pi_{it}^{j\delta}NW_{it}^{\mathrm{lv}}}{\chi_{it}^{j\delta}}\cdot f_{j\delta}'\left(p_t^j\right) \tag{17}$$

At this point it is helpful to bring in two concepts from the options pricing literature and practice, delta (Δ) and gamma (Γ). Delta and gamma will be the building blocks of our stability analysis. For each investor i and asset j, her Δ_{it}^j is the change in her net worth for every dollar increase in the price of asset j. That is:

$$\Delta_{it}^j\equiv\frac{\partial NW_{it}^{\mathrm{lv}}}{\partial p_t^j} \tag{18}$$

To get an explicit expression for Δ_{it}^j, we write out the investor's net worth in current period as her net worth last period, plus her profit from owning assets, plus her profit from the change in price of her derivative contracts:

$$NW_{it}=NW_{i,t-1}+(\text{Profit from owning asset})+(\text{Profit from derivatives}) \tag{19}$$

$$=NW_{i,t-1}+\sum_j m_{i,t-1}^{jd}\left(p_t^j-p_{t-1}^j\right)+\sum_{j\delta}C_{i,t-1}^{j\delta}\left[f_{j\delta}\left(p_t^j\right)-f_{j\delta}\left(p_{t-1}^j\right)\right] \tag{20}$$

[5] In general, using actual rather than minimum margin requirement would bias the estimate of MinMaSS downward and thus lead to insufficiently conservative policy, because higher minimum capital leads to lower MinMaSS, *ceteris paribus*. The difference between the relevant and actual margin requirements comes from the observation that market data shows the "average" margins, but the decisions depend on the "marginal" margins.

[6] The "delta" refers to the change in the value of the market maker's derivative position for a unit price change in the underlying the asset, here given by $C_{Ht}^{j\delta}f'(p_t^j)$.

Differentiating equation (20) with respect to the price of a specific asset j':

$$\Delta_{it}^{j'} = \frac{\partial NW_{it}}{\partial p_t^{j'}} \quad (21)$$

$$= m_{i,t-1}^{j'd} + \sum_\delta C_{i,t-1}^{j'\delta} f'_{j'\delta}(p_t^{j'}) \quad (22)$$

$$\approx m_{i,t-1}^{j'd} + \sum_\delta C_{i,t-1}^{j'\delta} \cdot \left[f'_{j'\delta}(p_{t-1}^{j'}) + f''_{j'\delta}(p_{t-1}^{j'})\left(p_t^{j'} - p_{t-1}^{j'}\right)\right] \quad (23)$$

$$= m_{i,t-1}^{j'} \quad (24)$$

Not coincidentally, the investor's $\Delta_{it}^{j'}$ is her total net effective demand for the asset.

We will also be interested in gamma (Γ). For each investor i and asset j, Γ_{it}^j measures the change in the investor's exposure to the asset as its price changes, assuming she does not actively adjust her positioning. It represents the convexity of her net worth relative to the price of asset j. By definition gamma is the price derivative of delta:

$$\Gamma_{it}^j \equiv \frac{\partial \Delta_{it}^j}{\partial p_t^j} \quad (25)$$

$$= \sum_\delta \frac{\pi_{i,t-1}^{j\delta} NW_{i,t-1}^{\text{lv}}}{\chi_{i,t-1}^{j\delta}} \cdot f''_{j\delta}(p_{t-1}^j) \quad (26)$$

These definitions of delta and gamma are analogous to those in the options literature (e.g., Hull, 2006).

3.1. Stability analysis

As with our previous analyses, the market for each asset j will be stable if demand is downward sloping. As before, we add fully funded investors who invest only in the cash market with a downward sloping demand curve:

$$m_t^{j,\text{ff}} = (1-\mu)ND_j(p_t^j) \quad (27)$$

The total demand curve is just the sum of the demand of all the investors:

$$m_t^{j,TOT} = \sum_i m_{it}^j + m_t^{j,\text{ff}} \quad (28)$$

The slope of the demand curve is:

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \frac{dm_{it}^j}{dp_t^j} + (1-\mu)ND'_j(p_t^j) \quad (29)$$

$$= \sum_i \frac{dm_{it}^j}{dp_t^j} - \frac{\eta_j A_j}{p_t^{j2}} \quad (30)$$

where η_j (defined as $-p_t^j D_j^{'}(p_t^j)/D_j(p_t^j)$, a positive number) is the elasticity of demand of fully funded investors and A_j (defined as $(1-\mu)Np_t^j D_j(p_t^j)$) is the total value of the assets they hold, as before.

Expanding the total derivative in terms of partial derivatives, and conducting a number of transformations (which are described in detail in Appendix B), gives the slope of the demand curve in terms of Δ and Γ as:

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[-\frac{m_{it}^j}{p_t^j} + \Gamma_{i,t+1}^j + \frac{\Delta_{i,t+1}^j \Delta_{it}^j}{NW_{it}^{\text{lv}}} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}} \frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j} \right] - \frac{\eta_j A}{p_t^{j2}} \quad (31)$$

Note that while some of the subscripts in this equation have the value $t+1$, these values are nonetheless all known at time t.[7]

To find MinMaSS and evaluate the stability of an equilibrium, we will be interested in the sign of this derivative in steady state, *i.e.* when $p_{t+1}^j = p_t^j$. In other words, if p_t^j is an equilibrium, is it stable?

The condition for stability in asset market j is that $dm^{TOT}/dp < 0$. Imposing this condition, dropping the now superfluous subscripts j and t, and rearranging terms gives another form of the stability condition:

$$\sum_i NW_i^{\text{lv}} \cdot \left(p\Delta_i / NW_i^{\text{lv}}\right)^2 + p^2 \left\{ \sum_i \Gamma_i + \sum_{\delta i} \frac{\partial m_{it}^j}{\partial \pi_i^\delta} \frac{\partial \pi_i^\delta}{\partial p} \right\} + (1-\eta)A < \sum_i pm_i + A \quad (32)$$

We define the left-hand side of equation (32) as MinMaSS; the right side is the actual market size, given by the total assets of the levered investors, plus total assets of the fully funded investors.[8] The four terms on the left side of equation (32) determine the minimum market size for stability.

In the first term, each investor makes a contribution to MinMaSS that is proportional to the square of her Δ relative to her net worth. Recall that Δ tells us the dollar amount that an investor's net worth changes for each dollar change in the price of an asset. The expression $p\Delta/NW^{\text{lv}}$ is therefore a measure of leverage. Indeed, in the simple case where there are no derivative contracts and an investor is invested in only one asset, $p\Delta/NW^{\text{lv}}$ is precisely equal to the conventionally defined leverage ratio.

Equation (32) therefore shows that each levered investor makes a contribution to MinMaSS in proportion to her net worth and the *square* of her leverage. This non-linearity means that a single investor can have a large impact on the market. The fact that contribution to MinMaSS is proportional to delta squared also means that a levered investor always makes a destabilizing contribution, whether she is long or short. If two levered investors enter into a futures contract, taking offsetting positions, both investors increase their squared delta and thus both contribute positively to MinMaSS. MinMaSS and instability increase with the total absolute value of levered investors' positions, not their aggregated net position, meaning that derivatives used for speculative purposes increase instability in proportion to the *square* of the net open interest in the contract.

This term also contains a contagion effect. If the price of another asset falls, so that the net worth of an investor decreases while Δ^j stays constant, the first term in equation (32) will increase, MinMaSS will rise, and the market will move closer to instability.

The second term of equation (32) is more subtle, but it will be intuitive to derivatives market participants as representing purchases or sales by market markers as a part of their delta hedging activities. This term arises because the value of a derivative in general will not be linear in the price of the underlying asset (long options positions have positive convexity, for example). As a result, if the price of the underlying asset changes, $f'(p)$ will change, and the market maker will no longer have a neutral stance with respect to the price of the asset (that is, equation (15) will no longer be satisfied). The market maker will therefore have to adjust her position in the underlying asset to remain hedged. This change in hedging demand in response to price is what is described by

[7] Mathematically, this can be seen in Appendix B in the transformation from equation (39) to (40).

[8] Equation (32) gives the stability condition where the independent variable is the price of an asset. However, many fundamentals-based investors consider relative value in their asset allocation decision, so that their demand for an asset depends not only on the price p^j of asset j but also on the price level of assets generally. It is easy to incorporate this into equation (32) via a change of variable. For example, we might let P be the general level of asset prices and $q^j = p^j - P$ be the idiosyncratic component of the price of asset j. Then $\partial/\partial q^j = \partial/\partial p^j$, so the equation does not change, but on the right side the elasticity η has a slight definitional change, and becomes: $\eta = \frac{(q^j+P)\cdot[\partial D^j(P,q^j)/\partial q^j]}{D^j(P,q^j)}$

the second term, and it shows that levered investors contribute to instability and MinMaSS in proportion to their net gamma position.

Because gamma only arises in relation to open derivative contracts, the total gamma in the market is zero.[9] However, we need to exclude from our calculation the gamma position of market makers, so this term will not in general be zero, although it is likely to be quite small compared to the first term. This leads to the interesting prediction that when levered investors sell volatility, stability in the underlying market is enhanced.[10]

The third term in equation (32) captures the contribution to MinMaSS from levered investors rebalancing their portfolios in response to price movements. In practice, this term is likely to be negative, though this is not certain; sufficiently inelastic substitution away from appreciating assets can cause levered investors actually to increase the proportion of net worth held in an asset as its price rises.

The fourth term in equation (32) is the elasticity-adjusted amount unlevered investors hold of the asset. This term is negative for $\eta > 1$. In this case, the greater share of assets held by fully funded investors, the lower is MinMaSS and the more stable is the market, all else equal.[11]

Each levered investor contributes to instability in proportion to the square of her leverage (Δ) and in proportion to her net volatility position (Γ). In practice, all the information necessary to evaluate the stability condition defined by equation (32) could be collected by a systemic regulator and straightforwardly aggregated to evaluate the stability of the market. While such an undertaking may sound daunting, in fact the information is simply a standard set of summary statistics of the portfolios of investors and is already compiled daily (or even more frequently) by all sophisticated investors in their risk reports.

Previous literature has measured financial stability empirically via market based measures such as CoVaR (Adrian and Brunnermeier, 2016), asset quality (Kamada and Nasu, March 2010) and the credit-to-GDP ratio (Borio and Lowe, 2002), among other metrics. Relative to that research, our approach is less complex, more transparent and tractable about what is driving the results, and hence straightforward to implement empirically. In the next section we argue that MinMaSS is a complementary metric to consider by applying the metric to the 1998 collapse of the hedge fund Long-Term Capital Management.

4. The collapse of long-term capital management

We now apply the model to the 1998 collapse of hedge fund Long-Term Capital Management and show that it risked destabilising those markets where it was both highly levered and relatively large. A number of previous authors have examined the near-collapse of LTCM, including Perold (1999), Jorion (2000), Schnabel and Shin (2004), and Dungey et al. (2006)).

Table 1
LTCM Performance and Leverage Ratio, Excluding Derivatives.

Date	Beginning Assets underManagement (Net Capital)	Annualized Return	End of Period Leverage(Excluding Derivatives)
3/94-2/95	$1.1 billion	25%	16.7
3/95-2/96	$1.8	50%	27.9
3/96-2/97	$4.1	34%	27.9
3/97-2/98	$5.8	11.5%	26.8
3/98-7/98	$4.7	-35%	31.0

Source: Perold (1999)

LTCM was a large relative value hedge fund that was in business from 1994 until 1999.[12] The hedge fund employed a strategy of *relative value arbitrage*, in which it bought some assets it considered to be relatively cheap while selling short other, very similar assets it considered to be relatively expensive.[13] Relative value hedge funds are typically highly levered institutions, and LTCM was perhaps the archetype of such a fund. LTCM was also very large: at its peak in April 1998 it had $4.87 billion in capital, $125 billion in assets, and another $115-125 billion of net notional value in off-balance sheet derivatives.[14]

For the first several years of its existence, LTCM's results were spectacular, and the fund grew as it succeeded. However, as it grew it attracted imitators both in the hedge fund community and among the trading desks of the Wall Street banks, leading returns and opportunities to dwindle (see Table 1). LTCM's troubles began in late spring of 1998 and continued into the summer 1998, especially when Salomon Brothers, whose trading behavior was close to that of LTCM, began to close down its arbitrage desk, both to reduce risk and in response to poor results.

In August and September, LTCM began to lose money in a dramatic fashion. However, LTCM's principals found themselves unable to liquidate to reduce risk at anything close to what they viewed as a reasonable price. Other market participants moved to liquidate ahead of LTCM, pushing prices against it and causing even deeper distress (Perold, 1999; Lowenstein, 2000).[15]

By the end of September, LTCM had barely been able to reduce its risk at all and its capital had been severely depleted. The Federal Reserve, cognizant that a default could result in a sudden liquidation of a portfolio that included $125 billion in assets and $1.25 trillion gross notional value of derivatives, and that this could destabilize markets,

[9] Recall that if the value of an investors' portfolio is a function $v(p)$ of the underlying price of the asset, then $\Gamma = v''(p)$. If the investor holds only outright long or short positions and has not entered into derivatives contracts, then the value of her portfolio is simply proportional to the price p, and $\Gamma = v''(p) = 0$.

[10] This phenomenon was confirmed in a conversation with a market participant who managed a large portion of the derivatives portfolio: he told us that when a large investor writes options, market makers delta hedging their positions are forced to buy when the market falls and sell when it rises, reducing volatility.

[11] In general, however, market stability increases with higher share of fully funded investors for *any* positive η. Consider equation (32) rewritten in terms of price elasticities for a single asset as follows:$(1-\epsilon)pm^{\mathrm{lv}} + (1-\eta)pm^{\mathrm{ff}} < pm^{\mathrm{lv}} + pm^{\mathrm{ff}}$ which simplifies to $\epsilon pm^{\mathrm{lv}} + \eta pm^{\mathrm{ff}} > 0$ and reflects the following stability condition: The elasticity of fully funded investors (η), weighted by their share of the market, outweighs the negative elasticity of the levered investors (ϵ), weighted by their share of the market. We are grateful to an anonymous Referee for this observation.

[12] The historical facts in this narrative are taken largely from Lowenstein (2000), Dunbar (2000), and MacKenzie (2003), as well as conversations with former LTCM principals who wish to remain anonymous.

[13] An example of a typical trade would be for LTCM to buy a 29 1/2-year Treasury bond and sell short a 30-year Treasury bond. LTCM made money because the 30-year bond was more liquid, so it traded with a slightly lower yield (i.e., higher price). After six months, the Treasury would issue a new 30-year bond, and the 30-year bond LTCM had sold short would become a 29 1/2-year bond while the 29 1/2-year bond it owned would become a 29-year bond. Because the 29 1/2-year bond has similar liquidity characteristics to the 29-year bond, the yields would converge, and LTCM could liquidate the trade at a profit. More examples of LTCM's trades can be found in Perold (1999).

[14] Other sources generally report a figure of $1.25 trillion in off-balance sheet derivatives. However, this figure does not net out long and short positions in the same instrument. The so-called "replacement value" of these swaps was $80-90 billion (Interview with LTCM Principal, 2010), and equity derivatives accounted for an additional $35 billion (Dunbar, 2000).

[15] Partner Eric Rosenfeld compared LTCM to a large ship in a small harbor in a storm—it was too large to maneuver, and all the other boats were just trying to get out of its way (Rosenfeld, 2009).

stepped in to orchestrate a bailout by LTCM's counterparties.[16]

We will never know for certain whether fears of a severe financial disruption would have been realized had LTCM been allowed to fail. We can, however, examine how the MinMaSS framework could have been used to assess whether financial markets would have suffered from an episode of instability had LTCM been forced to liquidate. We shall find that the instability might have occurred in at least a few of the markets in which Long-Term played.

4.1. Stability of LTCM's markets

We now examine LTCM's impact on the stability of global equity markets, global equity volatility markets, US bank funding markets and US Treasury markets in the late summer of 1998.[17] Bank funding and equity markets are two of the most economically significant and transparent markets in which LTCM operated, and accounted for a significant portion of the fund's risk. Furthermore, the bank funding market is of particular systemic importance since a dysfunctional bank funding market may cause contagious bank failures.[18]

Hard portfolio data on LTCM and its competitors are very difficult to come by because there were no public reporting requirements and the funds were very secretive while they were trading. Because of the media scrutiny to which LTCM was subject after the crash, some of the partners in the fund were more forthcoming than they had been previously, and some information is available on its portfolio. This information has been compiled from a number of media and academic sources, as well as a discussion with former LTCM principals. Information on the size of markets has been compiled from public sources such as the flow of funds accounts from the Federal Reserve.[19]

LTCM's largest equity trades were sales of volatility on broad stock indexes in the US and Europe.[20] According to Dunbar (2000), by January 1998 LTCM's 5-year equity option position was about \$100 million per percentage point of volatility. Using the Black-Scholes option pricing formula, this implies that LTCM had written options with a notional value of about \$11.5 billion (see Appendix C.1 for the calculation). The consequences of this position for stability in the equity market can be examined utilizing equation (32). For stability of the equity market as a whole, the consequences of LTCM's distress were small. LTCM's hedges meant that it had no exposure to outright stock price movements—it had a Δ of zero. Its Γ, however, (or $p^2\Gamma$) was about \$10 billion (see Appendix C.2 for the calculation).

Plugging these figures into equation (32), assuming no changes in LTCM's portfolio distribution and assuming the elasticity of fully funded demand to be equal to one, gives a MinMaSS in the equity markets of \$10 billion. The US and European equity markets at the time were larger than this by a factor of over a thousand, implying the markets were stable. Clearly, a forced liquidation of LTCM was nowhere near enough to destabilize the equity markets. Indeed, while equity markets declined along with most risk assets during the summer of 1998, they never ceased to function in an orderly manner.

The market for equity volatility was affected, however. LTCM's sales of volatility were one of the trades that hurt the fund the most, with losses of \$1.3 billion. We can use again equation (32) to determine the stability of the market for volatility by calculating MinMasSS. We assume no changes in LTCM's asset allocation shares during a forced liquidation, volatility priced around 20% and \$100 million exposure per point of volatility.[21] This translates into LTCM having sold short \$2 billion of volatility.[22]

We now calculate the total market size. According to Dunbar (2000) and Lowenstein (2000), LTCM was responsible for about a quarter of the long-term volatility sales, while the investment banks were responsible for the rest, meaning that the demand for volatility on the part of unlevered investors, A, was about \$8 billion. This volatility was mainly sold to pension funds and unit trusts that had promised their owners a minimum rate of return.

Since *pm* is LTCM's position (negative \$2 billion), the actual market size is therefore \$6 billion, and the stability ratio, given by the ratio of actual market size to MinMaSS, simplifies to:

$$\text{Stability ratio} = \frac{1}{3}(5 - 4\eta) \tag{33}$$

Assuming the elasticity of unconstrained demand to be equal to one (i.e., $\eta = 1$), the stability ratio is 3 (Table 2). If the elasticity of demand of unlevered investors was less than 0.5, then MinMaSS would have been higher than \$6 billion and the market for volatility would have been unstable. This is not entirely implausible, because the volatility was sold to insurance companies and pension funds that were using it to hedge guaranteed returns on their policies and likely would not have been inclined to sell their options to take advantage of short-term price movements. This shows that LTCM was of the right order of magnitude to destabilize this market.

A similar analysis is possible for another of LTCM's trades, a bet on swap spreads, a measure of bank funding costs. Comparison across sources suggests that LTCM's exposure to US swap spreads in the late summer of 1998 was about \$16 million per basis point of swap spread.[23] This corresponds to an exposure of about \$200 million per point of the price of a 10-year bond (MacKenzie, 2003), and therefore a notional exposure of \$20 billion both to Treasury bonds and to bank credit.

For the bank funding market we have LTCM's position ($pm =$ \$20 billion) and assets held by other investors ($A =$ \$618 billion).[24] Equation (32) implies a MinMaSS of \$200 billion, if the elasticity of unconstrained demand was equal to one. The associated stability ratio is about 3.2. However, if the demand for bank credit was inelastic ($\eta <$ about 1/3), then MinMaSS would have moved up towards the market size and the

[16] As then-Chairman Alan Greenspan put it, "our sense was that the consequences of a fire sale...should LTCM fail on some of its obligations, risked a severe drying up of market liquidity." (Greenspan, 1998) New York Fed President McDonough said, "there was a likelihood that a number of credit and interest rate markets would experience extreme price moves and possibly cease to function for a period of one or more days and maybe longer." (McDonough, 1998)

[17] While other funds and banks had made similar trades, these funds are considered here as behaving like fully funded investors since they were generally not facing distress and forced liquidations.

[18] The choice of these markets has been informed by the positions LTCM held, and in some sense seems obvious. We can use the MinMaSS framework to examine the stability of any market segment we can define. However, the narrower the definition of the market, the more elastic is the demand of unconstrained investors, and thus the smaller is the minimum market size for stability. When market size is larger, the levered investor such as LTCM must control a larger overall proportion of the market to destabilize it.

[19] Data for LTCM's competitors and imitators is even more scarce, as these entities (large investment banks and hedge funds) hold their proprietary trading data very closely. However, these funds tended to be both significantly smaller and less levered than LTCM (Anonymous, 1998), meaning that they contribute far less to MinMaSS. (Recall that that contributions to MinMaSS are proportional to fund capital and to the square of leverage.)

[20] More background on these trades, and the reasons behind them, are described in Perold (1999); Dunbar (2000); Lowenstein (2000).

[21] Lowenstein (2000) states that volatility was priced at 19%, while Perold (1999) cites a figure of 20%.

[22] See Appendix C.3 for the calculation.

[23] Lowenstein (2000, p. 187) implies that LTCM's exposure to a 15 basis point adverse move in swap spreads was \$240 million, which implies an exposure of \$16 million per basis point. Additionally, a former LTCM principal told us that \$10 million per basis point was a *plausible* estimate, which we take to mean that it is within a factor of two.

[24] Assets held by other investors is the sum of \$188.6 billion owed by commercial banks, \$193.5 billion owed by bank holding companies, \$212.4 billion owed by savings institutions, \$1.1 billion owed by credit unions, and \$42.5 billion owed by broker-dealers (Board, 2013), minus the \$20 billion held by LTCM.

Table 2
Stability Analysis for Selected LTCM Markets

	Equity Volatility	Bank Funding	US Treasury
LTCM Net Notional Exposure ($p\Delta$)	-$2 billion	$20 billion	$20 billion
LTCM Net Worth	$2.1 billion	$2.1 billion	$2.1 billion
Notional Position of Unconstrained Investors	$8 billion	$618 billion	$5.5 trillion
MinMaSS (Assumes $\eta = 1$)	$2 billion	$200 billion	$200 billion
Actual Market Size	$6 billion	$638 billion	$5.5 trillion
Stability ratio	3	3.19	27.5

stability ratio would have approached one. In this case, LTCM alone could have been enough to destabilize the market for bank credit.

By contrast, there were around $5.5 trillion of Treasury securities outstanding, which is around nine times the size of the bank funding markets. This leads to an stability ratio of more than 27, so LTCM likely was nowhere near big enough to destabilize the Treasury markets.

Real-world behavior is always more complex than economic models, but the narrative of the rise and fall of LTCM generally corroborates the key behavioral assumptions and predictions of the model. Appendix D provides an extended discussion of these real-world factors in light of the insights stemming from our model. The discussion shows that LTCM's behavior and financial market dynamics more closely match what is proposed in this paper or earlier models of capital-constrained arbitrage and investing behavior, such as Grossman and Vila (1992), Shleifer and Vishny (1997), or Liu and Longstaff (2004), than an unconstrained neoclassical agent. This is true in at least three important respects. First, the size of LTCM's positions during most of its existence was determined much more by its capital than by its assessment of available opportunities. Second, when the market tottered in the summer of 1998, the savviest investors were certain that prices were divorced from fundamental value, a view that was later proved correct. Third, prices in the destabilized markets became ill-defined as liquidity dried up.

5. Conclusion

This paper presents a simple quantitative framework to draw the link between leverage, market size, and financial stability. A market tends to become unstable when levered investors accumulate a large share of the assets in that market. The total net worth and the distribution of net worth held by levered investors together determine a minimum size for the market to be stable, MinMaSS. The ratio of the market size to this minimum market size defines a *stability ratio* which determines how close the market is to instability-induced crises.

Relative to the literature on leverage cycles (Brunnermeier and Pedersen, 2009; Adrian and Shin, 2014), our framework provides a particularly parsimonious model. We do not microfound the existence of leveraged investors, but simply ask what the presence of leveraged investors means for the stability of markets. Our resulting metric, the *stability ratio*, is straightforward to implement and measure empirically. This metric can thus be gauged by policy makers and market participants to understand the evolution of vulnerability in markets. Our empirical implementation falls squarely into the rapidly growing literature on systemic risk measurement (Duarte and Eisenbach, 2020; Acharya et al., 2012).

Our model is a simple quantitative approach to measuring the evolution of financial vulnerability. Our framework can accommodate the richness of financial instruments and heterogeneous beliefs, making it easy to apply operationally. The design implies robustness of the approach, and applicability across a range of markets and investor types. The key variables in the model are *observable* and *measurable* by regulators: leverage, margin requirements, interest rates, and the net worth of levered investors versus unlevered investors. The proposed framework does not rely on unobservable variables such as utility, expectations formation, and subjective probability distributions. The measure we present is sufficiently general and simple that it could be calculated and applied by macroprudential regulators to provide advance warning of a crisis, warnings that might prevent or mitigate future crises.

We apply the model to study the collapse of Long-Term Capital Management in 1998 and how the collapse affected various asset markets. Accounts of the demise of LTCM show that the fund was too highly levered. Our analysis makes clear that LTCM's leverage was only part of the story. LTCM was *both* highly levered *and* large relative to the markets in which it invested. Had it been smaller it might have survived even with its high leverage. LTCM's very existence destabilized markets, creating the potential for much larger price moves than would have been possible in the absence of the fund's existence. Instead of the risks being mitigated by LTCM's hedges, as it would have been had the fund been smaller, once the crisis hit risks became governed by the theoretical notional exposure. All of a sudden, the correlations between the assets LTCM was betting on changed, precisely because LTCM was betting on them.

CRediT authorship contribution statement

Tobias Adrian: Conceptualization, Writing – original draft, Supervision. **Karol Jan Borowiecki:** Writing – original draft, Validation, Software, Formal analysis. **Alexander Tepper:** Conceptualization, Writing – original draft, Formal analysis, Investigation, Resources.

Appendix A. Downward Sloping Demand and Investor's Net Worth

We assume that demand is downward sloping ($D^{'}(p) < 0$) and that it does not depend upon the investor's net worth. This is an abstraction which is unlikely to be fully correct in the real world. However, the dependence on price can capture this effect at each moment in time, since we have not specified a functional form. At first glance, it might appear that wealth effects could be large enough that they make fully funded investors' demand upward sloping. Closer examination reveals that this is not the case. Suppose that unlevered investors hold a proportion of their wealth $\beta(p)$ in an asset, and that they have net worth y. Then the number of units of the asset each fully funded investor demands is given by:

$$D(p) = \frac{y\beta(p)}{p} \tag{34}$$

Differentiating in logs gives:

$$\frac{d\log D(p)}{dp} = \frac{1}{y}\frac{dy}{dp} - \frac{1}{p} + \frac{\beta^{'}(p)}{\beta(p)} \tag{35}$$

The last term in this equation is negative. After examining the first term, we shall see that it is always outweighed by the second term, so that demand remains downward sloping. Suppose that the unlevered investor owns m assets at price p, and has additional assets y_0. Then

$$y = pm + y_0$$

and

$$\frac{1}{y}\frac{dy}{dp} = \frac{m}{pm + y_0} < \frac{1}{p}$$

for $y_0 > 0$. So the demand curve remains downward sloping.

Appendix B. Deriving the Stability Condition

Expanding the total derivative from equation (30) in terms of partial derivatives, we obtain:

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[\frac{\partial m_{it}^j}{\partial p_t^j} + \frac{\partial m_{it}^j}{\partial NW_{it}^{\text{lv}}}\frac{\partial NW_{it}^{\text{lv}}}{\partial p_t^j} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}}\frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j}\right] - \frac{\eta_j A}{p_t^{j2}} \tag{36}$$

We will aim to rewrite this equation in terms of Δ and Γ. Working just with the first term, we have:

$$\frac{\partial m_{it}^j}{\partial p_t^j} = \frac{\partial}{\partial p_t^j}\left[\frac{\pi_{it}^j NW_{it}^{\text{lv}}}{\lambda_i^j p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}^{\text{lv}}}{\chi_{it}^{j\delta}} \cdot f_{j\delta}'(p_t^j)\right] \tag{37}$$

$$= -\frac{\pi_{it}^j NW_{it}^{\text{lv}}}{\lambda_i^j p_t^{j2}} - \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}^{\text{lv}}}{\chi_{it}^{j\delta 2}} \cdot f_{j\delta}'(p_t^j)\frac{\partial \chi_{it}^{j\delta}}{\partial p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}^{\text{lv}}}{\chi_{it}^{j\delta}} \cdot f_{j\delta}''(p_t^j) \tag{38}$$

If the initial margin on derivative contracts χ is proportional to the price, as is usual, then we can simplify further:

$$\frac{\partial m_{it}^j}{\partial p_t^j} = -\frac{1}{p_t^j}\left[\frac{\pi_{it}^j NW_{it}^{\text{lv}}}{\lambda_i^j p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}^{\text{lv}}}{\chi_{it}^{j\delta}} \cdot f_{j\delta}'(p_t^j)\right] + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}^{\text{lv}}}{\chi_{it}^{j\delta}} \cdot f_{j\delta}''(p_t^j) \tag{39}$$

Substituting equation (17) in the first term and equation (26) in the second term gives:

$$\frac{\partial m_{it}^j}{\partial p_t^j} = -\frac{1}{p_t^j}\left[m_{it}^j\right] + \Gamma_{i,t+1}^j \tag{40}$$

We can now substitute equation (40) into the first term in the demand curve (36):

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[-\frac{m_{it}^j}{p_t^j} + \Gamma_{i,t+1}^j + \frac{\partial m_{it}^j}{\partial NW_{it}^{\text{lv}}}\frac{\partial NW_{it}^{\text{lv}}}{\partial p_t^j} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}}\frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j}\right] - \frac{\eta_j A}{p_t^{j2}} \tag{41}$$

Working now with the third term, we can substitute equation (18) to give:

$$\frac{\partial m_{it}^j}{\partial NW_{it}^{\text{lv}}}\frac{\partial NW_{it}^{\text{lv}}}{\partial p_t^j} = \left[\frac{\pi_{it}^j}{(1-\lambda_i^j)p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta}}{\chi_i^{j\delta}} \cdot f_{j\delta}'(p_t^j)\right] \cdot \Delta_{it}^j \tag{42}$$

$$= \left[\frac{m_{it}^j}{NW_{it}^{\text{lv}}}\right] \cdot \Delta_{it}^j \qquad \text{by (17)} \tag{43}$$

$$= \left[\frac{\Delta_{i,t+1}^j}{NW_{it}^{\text{lv}}}\right] \cdot \Delta_{it}^j \tag{44}$$

Substituting (44) into (41) gives the slope of the demand curve in terms of Δ and Γ, as expressed below, or in equation (31).

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[-\frac{m_{it}^j}{p_t^j} + \Gamma_{i,t+1}^j + \frac{\Delta_{i,t+1}^j \Delta_{it}^j}{NW_{it}^{\text{lv}}} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}}\frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j}\right] - \frac{\eta_j A}{p_t^{j2}} \tag{45}$$

Appendix C. Calculations

C1. LTCM's notional value of the outstanding options

An investor's exposure to changes in volatility is denoted by the Greek letter ν, referred to by options traders as vega. Vega is the derivative of the option price with respect to the implied volatility, or annualized standard deviation of the price of the underlying instrument. LTCM's 5-year equity option position was about \$100 million per percentage point of volatility (Dunbar, 2000), which implies a vega of \$10 billion. In standard option-pricing notation (see, for example, Hull, 2006), the vega of a put or call option is given by:

$$\nu = S_0\sqrt{T}N'(d_1) \tag{46}$$

where

- S_0 is the price (or for a portfolio, total notional value) of the underlying asset
- T is the time to expiry
- $N'(d_1)$ is the probability density function for the standard normal distribution, $N'(d_1) = \frac{1}{2\pi}\exp(-d_1^2/2)$
- $d_1 = \frac{\ln(S_0/K)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$
- K is the strike price of the option
- r is the risk-free rate of interest
- q is the dividend yield on the asset
- σ is the implied volatility, in percentage points per year

LTCM traded at-the-money-forward options, which means that $\ln(S_0/K) = -(r-q)T$, simplifying the expression for d_1. LTCM's vega is thus given by:

$$\begin{aligned} \nu &= S_0\sqrt{T}N'\left(\sigma\sqrt{T}/2\right) \\ (\$10\ \text{billion}) &= S_0\cdot\sqrt{5}\cdot\frac{1}{2\pi}\exp\left[-(20\%)^2\cdot 5/8\right] \end{aligned}$$

Solving for S_0, the notional value of the outstanding options, thus gives $S_0 = \$11.5$ billion.

C2. LTCM's Γ in the equity markets

Recall that Γ is the derivative of Δ with respect to the price. The Γ of an option is given by Hull (2006) as:

$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}} \tag{47}$$

where the variables are as defined as in Appendix C.1. We care about the $p^2\Gamma$ of the portfolio. If the portfolio includes options on m shares of the underlying asset, then the total $p^2\Gamma$ in equation (32) is:

$$\begin{aligned} p^2 m\Gamma &= p^2 m\frac{N'(d_1)}{S_0\sigma\sqrt{T}} \\ &= (p/S_0)\cdot pm\cdot\frac{N'(d_1)}{S_0\sigma\sqrt{T}} \\ &= 1\cdot(\$11.5\text{billion})\cdot\frac{N'\left(20\%\cdot\sqrt{5}/2\right)}{20\%\sqrt{5}} \\ &= \$10\ \text{billion} \end{aligned}$$

C3. MinMaSS of the market for equity volatility

MinMaSS is calculated using the following parameters:

- $\Delta = \$100$ million per volatility point
- $p = 20$ points
- $NW^{lv} = \$2.1$ billion
- $\Gamma = 0$

Note that while LTCM's equity in early summer was \$4.5 billion, only \$2.1 billion of this was actually required as margin for trades (MacKenzie, 2003), with the rest as risk capital intended to absorb losses. By early September, LTCM had lost its entire risk capital cushion. The \$2.1 billion is therefore the correct number to consider for the net worth as it was here that liquidations would have been forced. We again assume no changes in LTCM's portfolio balance during a forced liquidation and utilize the left-hand side of equation (32):

$$\text{MinMaSS} = p^2\cdot\left[NW^{lv}\cdot\left(\frac{\Delta}{NW^{lv}}\right)^2\right] \tag{48}$$

$$= (20)^2 \cdot \left[\$2.1b \cdot \left(\frac{-\$100m}{\$2.1b}\right)^2\right] \quad (49)$$

$$= \$2 \text{ billion} \quad (50)$$

It is clear from even a casual glance at LTCM's assets and equity (which Perold (1999) has obtained directly from LTCM, Figure 1) that the fund's equity is an important determinant of its assets. As LTCM ramped up its operations and began both to raise and earn capital, it was able to increase its assets as well, although, as Perold points out, the growth in assets outpaced the growth in capital for a time as LTCM built its operations. In fact, LTCM's balance sheet size was, in the early stages of its existence, more governed by right-sizing its assets to meet its capital base than by its assessments of the available arbitrage opportunities in the marketplace.

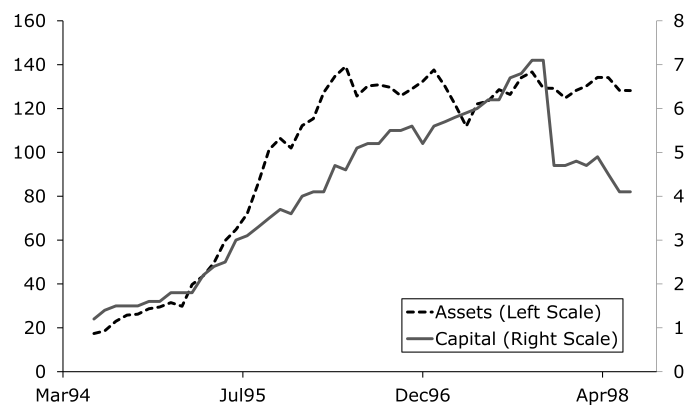

Fig. 1. LTCM Assets and Equity Capital Source: Perold (1999)

Appendix D. Corroborating Evidence

LTCM was not completely price-insensitive in its buying, contrary to what the model assumes about levered investors. Instead, as it grew and was imitated, LTCM noticed that favorable opportunities were "drying up big," as Eric Rosenfeld put it (quoted in MacKenzie, 2003). LTCM reacted by returning capital to investors, putting a ceiling on its willingness to continue its strategy at less and less attractive pricing. This decision to return capital to investors kept LTCM's capital at a level where margin constraints had to be considered. Instability was thus still very much a possibility because, as all the narrative histories of LTCM that we are aware of point out, capital cannot be easily raised by a fund once it begins to lose money and approaches its credit or net worth constraint.

This instability was realized in the summer of 1998 when prices began to move against LTCM. Even at the time, sophisticated market participants understood and commented that asset prices were moving away from fundamental value. As William Winters, head of J.P. Morgan's European Fixed Income business, put it at the height of the crisis, "any concept of long-term or fundamental value disappeared" (Coy and Woolley, 1998).

Other bankers agreed that prices were not being driven by fundamental value but by tactical considerations. As one Goldman Sachs trader put it, "If you think a gorilla has to sell, then you sure want to sell first." Goldman CEO Jon Corzine did not deny that the firm "did things in markets that might have ended up hurting LTCM. We had to protect our positions. That part I'm not apologetic about." (Lowenstein, 2000, p. 175) Lowenstein cites similar sentiments from executives at other banks, in particular Salomon Brothers, which had a portfolio of a similar size to LTCM (Dunbar, 2000).

In this situation, a value-oriented, unconstrained investor would be betting on prices to converge. So, too, would a levered investor who saw prices moving against her long-term view, and indeed, this is precisely what LTCM *wanted* to do. LTCM principals continued to have confidence in their trades even as the market moved further and further against them. As Meriwether put it in his August 1998 letter to investors (reproduced in Perold, 1999):

> With the large and rapid fall in our capital, steps have been taken to reduce risks now...On the other hand, we see great opportunities... The opportunity set in these trades at this time is believed to be among the best that LTCM has seen...LTCM thus believes it is prudent and opportunistic to increase the level of the Fund's capital to take full advantage of this unusually attractive environment.

Rosenfeld put it more directly: "We dreamed of the day when we'd have opportunities like this" (Lowenstein, 2000, p. 166).

This was not just talk. No source disputes that Meriwether was actively trying to raise additional capital. As Lowenstein (2000), Dunbar (2000), and others note, the partners' faith in their trades was ultimately proven correct. The consortium of investment banks that took over the fund was left with double-digit returns one year later. Yet as markets were moving against it, creating more attractive opportunities, LTCM was liquidating some trades, adding to the price pressure. According to Dunbar (2000, p. 194), LTCM decided at the end of June to reduce its daily value-at-risk (VAR) from $45 million to $35 million.[25]

[25] VAR is a measure of risk, typically quoted as the 95% confidence interval of daily profit and loss.

In the markets that LTCM destabilized, prices were not only divorced from fundamental value but in some cases were not even well-defined as liquidity evaporated. The stability ratios presented in Table 2 indicate that LTCM was of the right order of magnitude to destabilize the equity volatility market, but not the equity market. Liquidity conditions during the late summer of 1998 corroborate this finding. Despite significant declines, cash equity markets continued to function normally with reasonable liquidity. The market for long-dated volatility in equities, however, was thrown into disarray and became almost completely illiquid. Trading became very sparse and price quotes spiked and became divorced from fundamentals, according to market participants quoted in Dunbar (2000) and MacKenzie (2003).

As one banker said:

> When it became apparent that they [LTCM] were having difficulties, we thought that if they are going to default, we're going to be short a hell of a lot of volatility. So we'd rather be short at 40 than 30, right? So it was clearly in our interest to mark at as high a volatility as possible. That's why everybody pushed the volatility against them, which contributed to their demise in the end. (MacKenzie, 2003)

This quotation demonstrates that in the long-dated volatility markets, prices were not clearly defined. LTCM's counterparties had considerable discretion in what prices to place on the options that LTCM was short. This is only possible in a market that is not liquid. If the market had been active with many participants ready to buy and sell at their estimate of fundamental value, such discretion would not be possible because prices would be determined by the intersection of supply and demand. LTCM was such a big player that the possibility of it being forced to liquidate was enough to prevent prices from being well-defined. This is a hallmark of instability.

Given that the market moves were driven by fear of instability, it is no surprise that LTCM's losses were concentrated in the areas where it had most destabilized markets. Lowenstein (2000, p. 234) provides a breakdown, showing that of the $4 billion lost by Long-Term in 1998, $1.6 billion was in swaps and another $1.3 billion in equity volatility. No other category of losses even tops $500 million.

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